**The Zombie Apocalypse**

Imagine a world suddenly thrust into chaos by a zombie outbreak. In this world, you're not just a survivor but a scientist, analysing the dynamics of the outbreak to predict its course and devise strategies to save humanity. This is the essence of the "Zombie Apocalypse" model, a complex system incorporating susceptible humans, infectious zombies, and recovered individuals. We embark on a journey to understand the underlying principles of system dynamics, differential equations, and phase portraits.

**The Model**

Our tale unfolds with three protagonists:

* **Susceptible Humans (*S*)**: The uninfected population at risk of turning into zombies.
* **Zombies (*Z*)**: Those infected, roaming the streets, spreading the contagion.
* **Recovered Individuals (*R*)**: A beacon of hope, these are the ones who have fought the infection and emerged immune.

The fate of our world is governed by the interactions between these groups, encapsulated in a set of differential equations:

* *dt/dS*​=−*βSZ*
* *dt/dZ*​=*βSZ*−*δZ*−*γZ*
* *dt/dR*​=*γZ*

Here, *β* represents the transmission rate, *δ* the elimination rate of zombies by humans, and *γ* the recovery rate of zombies back to humans.

**The Quest for Survival**

With the script provided on blackboard, you will plot vector fields, manipulate parameters, and visualize the outcomes of different scenarios.

**Questions**

1. **When do the humans survive?** Analyse the model to determine the conditions under which the human population can avoid being overrun by zombies. What combination of parameters (*β*, *δ*, *γ*) leads to a hopeful outcome?
2. **Predicting the Turning Point:** At what stage of the outbreak can you predict the fate of the human race? Consider the initial conditions and the rate at which the situation evolves. How does the introduction of a recovery rate (*γ*) alter your predictions?
3. **Strategies for Survival:** Based on your analysis, what strategies would you recommend to maximize human survival? How do changes in the recovery rate (*γ*) affect the overall dynamics of the outbreak?